Base Belief Revision for Finitary Monotonic Logics

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LoMoReVI

Outline

Logical Preliminaries

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5. Structural: if $\Gamma \vdash_{\mathcal{S}} \varphi$ then $h[\Gamma] \vdash_{\mathcal{S}} h(\varphi)$, for any homomorphism $h : \mathbf{Fm} \to \mathbf{Fm}$.

Belief States: the Objects of Change

A *belief state* of an agent is described by a set of sentences

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1. deductively closed sets (theories), where $T = \mathrm{Cn}_{\mathcal{S}}(T)$, and
2. arbitrary sets (bases), where condition $T \vdash_{\mathcal{S}} \varphi$ *implies* $\varphi \in T$ is dropped

Prioritized Change Operations

Given a belief state T and an input φ, *prioritized* change operations (enforcing change by input)

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. . . subject to condition: minimize changes in T (making further information loss logically unnecessary)

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$$T *^{T_0} \varphi = \begin{cases} T \circledast \varphi & \text{if } T_0 \cup \{\varphi\} \text{consistent} \\ T & \text{otherwise} \end{cases}$$

Partial Meet

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Other ranking-based methods: entrenchment relations $\prec$, system of spheres $\$$

Partial Meet (i) Tools: Unprovability of vs Consistency with

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1. $X \subseteq T$

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A *coherer* set $X \in \mathrm{Con}(T, \varphi)$ is:

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Partial meet (ii)-(iv): selection functions and meet

Definition

Given T_0, a *selection function for* T_0 is a function

$$\gamma : \mathcal{P}(\mathcal{P}(\mathbf{Fm})) \setminus \{\emptyset\} \longrightarrow \mathcal{P}(\mathcal{P}(\mathbf{Fm})) \setminus \{\emptyset\}$$

such that for each input $T_1 \subseteq \mathbf{Fm}$,

$$\emptyset \neq \gamma(\mathrm{Con}(T_0, T_1)) \subseteq \mathrm{Con}(T_0, T_1)$$

(and similarly for Rem)

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Partial meet operators (for T): let $\varphi \in \mathbf{Fm}$ be arbitrary:

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3. Con – Revision $\quad T \circledast_\gamma \varphi = (\bigcap \gamma(\mathrm{Con}(T, \varphi))) \cup \{\varphi\}$

The Deduction Theorem and a Duality Rem - Con

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Let $\mathcal{S}$ be a logic. We say $\mathcal{S}$ has the Deduction Theorem iff

$$T \cup \{\varphi\} \vdash_{\mathcal{S}} \psi \text{ iff } T \vdash_{\mathcal{S}} \varphi \to \psi$$

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Recall that Gödel Logic, i.e. BL + $\varphi \to (\varphi \wedge_* \varphi)$, satisfies the Deduction Theorem.

AGM Characterization of Theory Contraction

Definition

Axioms for **AGM**-contraction $\ominus$:

1. $T \ominus \varphi$ is a belief set (type)
2. $T \ominus \varphi \subseteq T$ (contraction)
3. If $\varphi \notin T$ then $T = T \ominus \varphi$ (min action)
4. If $\nvdash \varphi$ then $\varphi \notin T \ominus \phi$ (success)
5. If $\varphi \in T$ then $T \subseteq (T \ominus \varphi) \oplus \varphi$ (recovery)
6. If $\vdash \varphi \Leftrightarrow \psi$ then $T \ominus \varphi = T \ominus \psi$ (ext.)
7. $((T \ominus \varphi) \cap (T \ominus \psi)) \subseteq T \ominus (\varphi \wedge \psi)$ (min-conj.)
8. If $\varphi \notin T \ominus (\varphi \wedge \psi)$ then $T \ominus (\varphi \wedge \psi) \subseteq T \ominus \varphi$ (max-conj.)

Theorem

Let $\mathcal{S}$ be supraclassical with the Deduction Thm. An operator $\ominus : \mathrm{Cn}_{\mathcal{S}}[\mathcal{P}(\mathbf{Fm})] \times \mathbf{Fm} \longrightarrow \mathcal{P}(\mathbf{Fm})$ satisfies these axioms iff $\ominus = \ominus_\gamma$

AGM Characterization of Theory Revision

Definition

AGM postulates for revision $\circledast$:

1. $T \circledast \varphi$ is a theory (type)
2. $\varphi \in T \circledast \varphi$ (success)
3. $T \circledast \varphi \subseteq T \oplus \varphi$ (upper bound)
4. If $\neg\varphi \notin T$ then $T \oplus \varphi \subseteq T \circledast \phi$ (lower bound)
5. $T \circledast \varphi$ is consistent, if φ is (triviality)
6. If $\vdash \varphi \Leftrightarrow \psi$ then $T \circledast \varphi = T \circledast \psi$ (extensionality)
7. $T \circledast (\varphi \wedge \psi) \subseteq (T \circledast \varphi) \oplus \psi$ (iterated ($T \circledast 3$))
8. If $\neg\psi \notin T \circledast \varphi$ then $(T \circledast \varphi) \oplus \psi \subseteq T \circledast (\varphi \wedge \psi)$ (iterated ($T \circledast 4$))

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A comparison: Theory Change - Base Change(1 of 3)

Syntax-sensitivity

Base Change is Syntax-sensitive:

Example

$$\{p \wedge q\}$$

$$\{p, q\}$$

Example

$$\mathrm{Cn}(\{p \wedge q\})$$

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A disadvantage for Base Change?

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A disadvantage for Base Change? Base Revision might output suboptimal results, unless additional Base manipulation ($\wedge$-atomization) is performed.

A comparison: Theory Change - Base Change (2 of 3) Recovery

Theory Change satisfies Recovery:

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Theory Change satisfies Recovery:

Base Change does not recover (by default, but can be enforced to do so at will). An advantage for Base Change?

A comparison: Theory Change - Base Change(3 of 3) Maxichoice
The status of Maxichoice change processes and the behavior of Maxichoice operators.

Theorem

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The limiting case of Maxichoice is not a natural case in theory revision,

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$$\mathrm{Cn}(T) \circledast_{mc} \varphi \text{ is complete}$$

The limiting case of Maxichoice is not a natural case in theory revision,while in base revision it does capture epistemically optimal situations.

Hansson-Wassermann Characterization of Base Contraction

Definition

Axioms for Base Contraction:

- $\varphi \notin T \ominus \varphi$, if $\emptyset \nvdash_{\mathcal{S}} \varphi$ (success)
- $T \ominus \varphi \subseteq T$ (inclusion)
- If $\psi \in T \setminus T \ominus \varphi$ then there is T' with $T \ominus \varphi \subseteq T' \subseteq T$ such that $\varphi \notin T'$ but $\varphi \in T' \cup \{\psi\}$ (relevance)
- If for all subsets T' of T, $\varphi \in \mathrm{Cn}_{\mathcal{S}}(T')$ iff $\psi \in \mathrm{Cn}_{\mathcal{S}}(T')$ then $T \ominus \varphi = T \ominus \psi$ (uniformity)

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For each monotonic compact logic $\mathcal{S}$, $T \subseteq \mathbf{Fm}$ and $\varphi \in \mathbf{Fm}$.

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Theorem

*For each monotonic compact logic $\mathcal{S}$, $T \subseteq$ **Fm** and $\varphi \in$ **Fm**.*

$$\ominus \text{ satisfies these axioms iff } \ominus = \ominus_\gamma, \text{ for some } \gamma$$

Hansson-Wassermann Base Revision

For Base Revision, Hansson and Wassermann Rem-based characterization depends on a further condition upon $\mathcal{S}$:

Definition

φ-*non-contravention* is the property:

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Fact

Fm-*non-contravention holds in logics* $\mathcal{S}$ *with the Deduction Theorem and the structural axiom of Contraction.*

Definition

Axioms for (Internal) Revision.

If $\neg\varphi \notin \mathrm{Cn}_{\mathcal{S}}(\emptyset)$ then $\neg\varphi \notin \mathrm{Cn}_{\mathcal{S}}(T \widehat{\circledast} \psi)$ (non-contradiction)

1. $T \widehat{\circledast} \varphi \subseteq T \cup \{\varphi\}$ (inclusion)
2. If $\psi \in T \smallsetminus T \widehat{\circledast} \varphi$, then there is $T' \subseteq T$ such that $T \widehat{\circledast} \varphi \subseteq T' \subseteq T \cup \{\varphi\}$, $\neg\varphi \notin \mathrm{Cn}_{\mathcal{S}}(T)$ but $\neg\varphi \in \mathrm{Cn}_{\mathcal{S}}(T \cup \{\psi\})$ (relevance)
3. $\varphi \in T \widehat{\circledast} \varphi$ (success)
4. If for all $T' \subseteq T$, $\neg\varphi \in \mathrm{Cn}_{\mathcal{S}}(T')$ iff $\neg\psi \in \mathrm{Cn}_{\mathcal{S}}(T')$, then $T \widehat{\circledast} \varphi = T \widehat{\circledast} \psi$ (uniformity)

Theorem

Let $\mathcal{S}$ be a finitary and monotonic logic satisfying non-contravention, then $\widehat{\circledast}$ is a Rem*-based partial meet internal revision operator iff $\widehat{\circledast}$ satisfies the preceding axioms.*

Observe Gödel Logic *G* has both the Deduction Theorem and the

An example of non-contractive logic: Łuaksiewicz

Definition

The Łukasiewicz t-norm $*_{\text{Ł}}$ is defined by $r *_{\text{Ł}} s = \max\{0, r+s-1\}$. Its residuum $\Rightarrow_{\text{Ł}}$ is defined by $r \Rightarrow_{\text{Ł}} s = \min\{1, 1-r+s\}$. Łukasiewicz logic, Ł, is Hájek's **BL** extended with axiom $\neg\neg\varphi \rightarrow_{\text{Ł}} \varphi$. It interprets strong conjunction & by $*_{\text{Ł}}$ and conditional $\rightarrow_{\text{Ł}}$ by $\Rightarrow_{\text{Ł}}$. Logic Ł captures validities of t-norm $*_{\text{Ł}}$.

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This logic satisfies only the (weaker) *Local Deduction Theorem*:

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Dropping the Deduction Theorem condition for Revision.
Recall Con-based $\circledast_\gamma$ operators: $T_0 \circledast_\gamma T_1 = \bigcap \gamma(\mathrm{Con}(T_0, T_1)) \cup T_1$.

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Axioms for (multiple) base revision

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Limiting cases: Full Meet and Maxichoice

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$$T_0 \circledast_{\mathrm{fm}} T_1 = (\bigcap \mathrm{Con}(T_0, T_1)) \cup T_1 \quad \text{and } T_0 \circledast_{\mathrm{mc}} T_1 = X \cup T_1$$

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An example: **RPL** and $\mathbb{Q}$-closed bases.

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Rational Pavelka Logic is an extension of Ł with truth-constants $\overline{r}$ for each $r \in [0,1] \cap \mathbb{Q}$. Additional *book-keeping* axioms are: $\overline{r} \& \overline{s} \equiv \overline{r *_{\text{Ł}} s}$ and $\overline{r} \rightarrow_{\text{Ł}} \overline{s} \equiv \overline{r \Rightarrow_{\text{Ł}} s}$.

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Rational Pavelka Logic is an extension of Ł with truth-constants $\overline{r}$ for each $r \in [0,1] \cap \mathbb{Q}$. Additional *book-keeping* axioms are: $\overline{r} \& \overline{s} \equiv \overline{r *_{Ł} s}$ and $\overline{r} \rightarrow_{Ł} \overline{s} \equiv \overline{r \Rightarrow_{Ł} s}$.

We can distinguish between plain bases and $\mathbb{Q}$-closed bases:
$(\varphi, r) \in \mathrm{Cn}_{\mathbb{Q}}(T)iff(\varphi, s) \in T,$ for some $s \geq r$.

Fact

(Partial meet) Revision of a $\mathbb{Q}$-closed bases by a $\mathbb{Q}$-closed base is $\mathbb{Q}$-closed.

Example

(In **RPL**) Let $T = \{(p, 0.7)\}$. Then for input $\varphi = (\neg p, 0.4)$ we have

1. $T \circledast \varphi = \{(\neg p, 0.4)\}$, while
2. $\mathrm{Cn}_{\mathbb{Q}}(T) \circledast \varphi = \mathrm{Cn}_{\mathbb{Q}}(\{(p, 0.6), (\neg p, 0.4)\})$ (i.e. $p \equiv 0.6$)

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2. For monotonic logics, the result for contraction from Hansson and Wassermann seems optimal (unless we plan to use contraction to define revision)
3. But for revision, we can improve the current state of the art characterization result: the Deduction Theorem and the structural axiom of Contraction are not required.
4. E.g. one can safely revise Belief Bases under t-norm based Fuzzy logics, (some of) their expansions with truth-constants.

Thank you !!

Any questions ?